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Technical Report 5

**OPTIMAL TRAJECTORIES FOR AIRCRAFT TERRAIN FOLLOWING  
AND TERRAIN AVOIDANCE - A LITERATURE REVIEW UPDATE**

by

M.E. HALPERN

Approved for public release.

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OPTIMAL TRAJECTORIES FOR AIRCRAFT TERRAIN FOLLOWING  
AND TERRAIN AVOIDANCE - A LITERATURE REVIEW UPDATE

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M.E. HALPERN

SUMMARY

*This is a literature review relevant to the automated design of flight paths for low-flying military aircraft in terrain following and terrain avoidance roles. Improvements in computer technology continue to widen the range of approaches which may be applied to this important problem. In particular, there is interest in the application of techniques which use elements of artificial intelligence.*



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## 1. INTRODUCTION

The automated generation of flight paths for military aircraft is of interest for several reasons. Automated systems offer the potential to reduce the pilot's workload on the tasks currently performed. They also offer the possibility of introducing otherwise unfeasible features to the mission, such as in-flight replanning in response to the receipt of intelligence, rather than aborting a mission. Aspects of the trajectory design problem also apply to non-piloted and remotely piloted vehicles.

This work surveys the relevant literature in the area. Previous literature surveys in the general area have been carried out by Hill [1], who surveyed work on integrated control, which included a component related to trajectory generation, and by Waller [2], who examined the literature on aircraft trajectory generation for both military and civilian aircraft. The work presented here focusses on trajectory generation with a view to minimising risk to the military aircraft carrying out a mission. Here, risk may incorporate such components as proximity of path to threats and length of path.

As a result of recent developments in the area, a considerable amount of the work reviewed here uses techniques from artificial intelligence (AI) to aid in the design of flight paths. These algorithms are explained in this work, since it is considered likely that readers will be less familiar with the ideas underlying these techniques than they would be with the more established approaches.

It is probably important to state why optimal trajectories are being considered. It is not for the obvious reason that optimal trajectories are better than others in some sense, since, in practice, the quantities which can be optimised, or most readily optimised, are not necessarily related to operational requirements. The main reason optimal trajectories are considered is that in trajectory design, as in many other engineering design problems, formulating the design as an optimisation leads to a systematic framework for dealing with trade-offs in the design. In some instances, this framework may also provide insights about such issues as bounds on achievable performance and the sensitivity of the optimal result to variations in the problem setup. For many problems, there is also a desirable robustness result associated with optimal solutions.

## 2. TERRAIN FOLLOWING

The terrain following (TF) problem is concerned with choosing a flight path in a vertical plane only, over a prespecified straight ground track. The major objective is to choose a path which is as close as possible to the ground whilst being safely flyable. The techniques used to design such paths fall into three categories:

- (a) optimisation using combinatorial search techniques,
- (b) geometric approaches to path design,
- and (c) design based on optimal control theory.

## 2.1 Combinatorial Search

The main technique used here is dynamic programming (DP), described, for example by Larson and Casti [3]. DP is an optimisation technique for finding a minimum cost path through a discretised grid. Effectively, it checks all of the possible paths to find the optimal one, but uses the Principle of Optimality, described in [3], to avoid redundant checking. The approach has the desirable feature of finding the global optimum, as opposed to local optima which some optimisation algorithms can obtain.

Another benefit of using DP is that, since it is a numerical optimisation which uses numerical values of cost functions, these are allowed to be of fairly general form. This flexibility allows the ready incorporation of features which may be difficult to implement using other approaches. An example of such a feature is a "soft" threat, that is a threat which may be penetrated, but with a penalty, in contrast to a "hard" threat which cannot be traversed.

A major problem with the approach is that the search space and thus the amount of computer memory and processing requirements can become very large if the problem has many dimensions or if the discretisation is very fine. A fine discretisation may be needed to approximate closely a continuous solution.

Aspects of applying this technique to the aircraft TF problem have been examined by Rigopoulos and co-workers in [4] and [5]. In those works, the path design problem was posed as finding the piecewise linear vertical profile which minimised a linear or quadratic measure of altitude above a clearance path, subject to bounds on allowable discretised slope, curvature and kink. A two step process, involving the design of a coarse path, followed by a fine grain path, restricted to a corridor about the coarse path, in order to reduce the computational requirements, was implemented and was found to give good quality solutions.

## 2.2 Geometric Methods

The main technique used here is the design of a path consisting of cubic splines. These are piecewise cubic curves which have continuous slopes and curvatures at their joints which are called knots. Cubic splines are useful as curves for aircraft trajectories because they can be designed with second derivatives (which, if the velocity is constant, and the path is close to horizontal, approximately correspond with normal accelerations in the TF problem) that are small and bounded. Work using this type of approach does not seem to be applied to "soft" obstacles.

Funk [6] has developed an approach for generating optimal cubic spline paths for the aircraft TF problem. These paths were optimal in the sense of minimising some measure of height above a ground clearance curve whilst not exceeding bounds on slope, second

and third derivatives (third derivative corresponds with jerk and has practical importance in the examination of G-induced loss of consciousness). In that work, the problem was formulated to allow the measure of height being minimised to be either a linear or a quadratic sum of heights at spline knots above a ground clearance curve.

Andmark and Wirkander [7] have used linear programming to design cubic spline paths which minimise a linear sum of heights of knots above a clearance curve. Their approach is similar to that of Funk [6] with a linear cost function, but they give more detail on the form of the equations to be solved.

In [8], Berrean and co-workers used a quadratic programming approach to design cubic splines for an aircraft TF application. That work was also based on that of Funk [6]. The work also contained some discussion of solutions obtained using DP and gave some indications on the relative advantages of the two methods.

### 2.3 Optimal Control Based Methods

Links between the design of control systems and the design of aircraft trajectories are strong. An important approach to the design of aircraft trajectories involves formulating the trajectory design problem as a control system design problem. This is appropriate because an aircraft is a dynamic system and control system design is concerned with dynamic systems. This approach is useful because it allows the trajectory designer to draw upon the large body of knowledge which has been amassed on control system design over the last few decades.

Halpern [9, 10] applied the theory of quadratically optimal tracking systems to the aircraft terrain following problem. In this work, the terrain profile was used as a command input for the controlled aircraft to follow. Smoothing of the trajectory was achieved by penalising elevator activity. An important feature of this application is that future values of the command signal, that is to say, terrain height values ahead of the aircraft, are available from a terrain database. It was demonstrated how this future information could be used to obtain improved performance, in the sense of achieving more accurate tracking of the terrain profile, thereby allowing a lower altitude flight path, while using less actuator activity. A limitation of that work was the aircraft dynamics were assumed to be linear, whereas, in fact, they are nonlinear. Another limitation is that the quantity minimised was such that positive and negative tracking errors were penalised identically. This does not reflect the seriousness of flying too low rather than too high. Nonetheless, much work on flight path control has been done using these simplifying assumptions and performance measures, for example, Jung and Hess [11] have applied a generalised predictive control algorithm to a terrain following rotorcraft.

### **3. TERRAIN FOLLOWING/ TERRAIN AVOIDANCE (TF/TA)**

This involves the design of a three (or four, if intercept times are considered) dimensional path. The important difference between this problem and the TF problem is that with TA, the ground track is to be determined. This complicates considerably the formulation of the problem. A conceptually straightforward extension of the TF problem would be to formulate the path design to encourage a path over low terrain. This would increase the dimension of the problem in a mathematical sense. However, there is also the possibility of choosing the path to make use of adjacent terrain to mask parts of the trajectory from specific threats.

This problem is thus much more difficult, both conceptually and computationally, than the TF problem. There is interest in the application of techniques from Artificial Intelligence (AI) to the problem at two distinct levels. The first of these uses knowledge based techniques to emulate human behaviour for activities such as feature recognition and high level planning. The second level is the use of specific algorithms embodying AI principles to design paths. These types of algorithms seem appropriate for this type of application since they are used to obtain suboptimal, yet satisfying solutions to large optimisation problems in a relatively small time. A satisfying solution to an optimisation problem is a solution which satisfies the constraints in the problem, but is not optimal and is usually found more easily than the optimal solution.

#### **3.1 Spline Based Methods**

Jackson and Crouch [12] have investigated the use of cubic splines to design four dimensional runway approach paths to make use of the Microwave Landing System. As well as investigating a purely static geometric approach, which involved interpolating specified waypoints, they examined the use of "dynamic interpolating splines". For this, they assumed a simple dynamic model for the aircraft and designed a path which passed through the waypoints while minimising a quadratic functional of the accelerations. Note that this work was not applied to terrain avoidance, but the techniques used are of interest and are relevant to aspects of that problem.

#### **3.2 Combinatorial Search**

##### **3.2.1 Dynamic Programming**

The Dynapath algorithm [13] was developed by the TAU Corporation and uses dynamic programming to generate an optimal flight path. The path is optimal in the sense that a weighted quadratic sum of lateral deviation from a reference path and altitude above a reference altitude is minimised. This minimisation is subject to bounds on clearances and accelerations. Two versions of the Dynapath algorithm were developed. The first of these directly generated three dimensional paths. The second separately designed the lateral and

vertical parts of the path. This was done in order to reduce the computational burden. There is always a possibility, with this type of approach, that the solution obtained may not be the true optimum, which is found by solving the larger problem.

Dorr [14], at NASA Ames, has used Dynapath in a simulation as part of the development of an automated Nap-of-the-Earth helicopter flight capability.

The Advanced Mission Planning System (AMPS) was foreshadowed in [15] as using DP in its route planner.

### 3.2.2 A\* Algorithm

An important alternative algorithm to dynamic programming is the *A\** (pronounced A star) algorithm, which is a heuristic shortest path algorithm. This has the potential to handle the same kinds of problems that D<sub>1</sub> does, but allows heuristic knowledge about the application to be incorporated, allowing the optimal solution to be found with less than the effectively exhaustive search carried out in DP.

The heuristic knowledge takes the form of an estimate of the optimal cost from a node in the search tree to the goal. The cost from the start to the node is calculated exactly. If the estimate of the optimal cost from the node to the goal is less than the true optimal cost from the node to the goal, the heuristic is said to be admissible and the search procedure will yield the optimal solution. The closer an admissible estimate is to the true optimal cost, the better informed is the heuristic and the smaller is the number of nodes expanded in the process. Thus there is a trade-off between the complexity of calculating a well informed heuristic and the necessity of expanding a large number of nodes if the heuristic is poor. The algorithm may be modified to give a near optimal solution in a shorter time. Methods for achieving this are discussed by Pearl [16] and he indicates that it is often worthwhile sacrificing a guarantee of optimality and settling for a satisficing solution.

An early application of the *A\** algorithm to the generation of an aircraft flight path was by Lizza and Lizza [17] in which simple two dimensional paths which minimised a cost function involving threat interaction and path length were obtained. The path generated consisted of straight legs. The lengths of the legs and the angles between adjacent legs were input by the user, but it is not stated how these were chosen. Nonetheless, despite shortcomings in relating this work to aircraft performance limitations, the work did demonstrate the computational advantages of *A\** over some other algorithms in this kind of application.

Pilot's Associate (PA) was a programme sponsored jointly by the Defense Advanced Research Projects Agency and the U.S. Air Force. The programme was intended to apply the technologies of real-time, co-operating knowledge based systems for exploring the potential of AI to improve mission effectiveness and survivability of advanced fighter

aircraft [18]. Lockheed and McDonnell were the main contractors involved in the later stages of the project. The Lockheed effort is described in [19] and the McDonnell perspective in [18]. Some of the work done by the McDonnell side is described here.

In [20], Bate and Stanley, whose goal was to apply advanced computer technologies to the problem of automated inflight aircraft mission planning, state that their work has been heavily influenced by interaction with the PA program. Although  $A^*$  would be expected to be computationally advantageous compared with DP, it is still necessary to reduce somehow the size of the search space for a practical real-time system based on current technology. In [20], this was achieved by choosing a mission planning architecture which decomposed the route planning hierarchically into two levels. The mission level route planner used  $A^*$  to design a coarse two dimensional flight path consisting of 2 km straight line segments. This path gave an optimal trade-off between path length and danger. Here, danger was obtained from a gridded danger map which contained threat information. The tactical level route planner used the  $A^*$  algorithm to design a section of refined path in a corridor containing the coarse path. This refined path consisted of constant turn radius segments, each representing one segment of flight. Connecting segments were allowed to have different radii, obtained by incrementing or decrementing a horizontal load factor. The cost function minimised by this path was also a trade-off between path length and danger, but here the danger component was more elaborate and took into account line of sight between the aircraft and threats.

The Knowledge Based Route Planning Study [21], also carried out by McDonnell Douglas, was to develop a system which automatically plans a route for a tactical aircraft on a low-altitude TF/TA mission. One aim of the study was to investigate if symbolic map segmentation and progressively deepening search were of practical use. The approach involved three phases. The first was to preprocess real data to reduce the search space to be operated on by the route planner in the second phase. Phase three was an interactive explanation facility. In the first phase, heuristic rules were used to identify features on the map and some flight corridors were identified. The route planning was then carried out by repeatedly using the  $A^*$  algorithm in a corridor using increasing levels of map detail. Such progressively deepening search has the advantage that it is potentially faster than conventional  $A^*$ , but has the disadvantage that the guarantee of optimality has been lost.

Work has also been carried out at ARL on the design of route planners using  $A^*$ . Goss and Selvestrel developed a concept demonstrating visualisation tool [22] and used it in a study [23, 24] of cost functions and computational bounds of admissible and inadmissible heuristics. That system generated trajectories which consisted of two dimensional straight line segments. These paths minimised a weighted sum of total path length and the sum of path segments weighted by the terrain altitude below each of them, in order to produce a trajectory which gave a trade-off between a short path length and a path which avoided high terrain.

Their work was used as a basis for the development by Halpern [25] of concepts for a more realistic planner which generated ground track segments including circular arcs. In that work, the search space was reduced by using moves of differing length. In regions where the path was less critical, that is where there were no waypoints, longer path segments were used, so that fewer nodes were expanded. This work also incorporated soft threats.

Pilots' knowledge was incorporated into the mission planner described by Rouse [26]. Their evaluations of the desirability of various scenarios were stored in a pattern classifier which then provided estimates of evaluations for previously unconsidered scenarios. These estimates were subsequently used to contribute to the cost function in an optimal trajectory generator using the A\* algorithm.

In [27], Barboza used object oriented programming techniques were used to give a dynamic environment model. This was done to provide more realistic interactions between threats and the penetrator, for example, threats could be considered to be in a state of readiness if a neighbouring threat had detected a penetrator. The A\* algorithm was used to find a lowest cost path.

From the work described in this Section, then, it may be seen that the use of A\* for aircraft trajectory generation is quite widespread.

### 3.3 Control Theoretic Methods

As mentioned previously, a criticism which may be levelled at the application of combinatorial search based optimisation approaches to discretised search spaces as discussed above is that the search spaces can be very large, particularly if fine discretisation is used. A fine discretisation may be required to give a smooth trajectory. An alternative formulation which avoids the problem is to optimise a continuous function over a continuous domain.

Asseo [28] used a method based on calculus of variations (see Bryson and Ho [29]) to minimise a quadratic cost function penalising large cross-track deviations from an initial ground track, altitude above sea level and proximity to known threats. The algorithm was used to generate two dimensional ground tracks which were combined with parabolic vertical profiles. It was stated that the approach was more computationally efficient than DP, but the complete algorithm was quite complicated and involved solving a two point boundary value problem within an iterative loop. It was also stated that a global optimum was not guaranteed to be found and various measures were taken to ensure convergence of the solution.

The use of Pontryagin's Maximum Principle [29, 30] to generate three dimensional helicopter trajectories is described by Menon and co-workers in [31]. A disadvantage of

the method was that first and second derivatives of the surface describing the terrain were required. This required some processing of the terrain data. The method proceeded by integrating a set of three nonlinear differential equations, inside an iterative framework involving a one dimensional search. The cost function minimised was a linear combination of flight time and a terrain masking function. A simple masking function which encouraged low flight was used.

Vian and Moore [32] at Boeing have used Pontryagin's Minimum Principle in conjunction with reduced order aircraft models obtained using singular perturbation theory to design lateral and vertical trajectories with fixed or free final time. The cost function minimised included components associated with time, fuel and distance from threats.

This work was extended by Rao and co-workers [33] to incorporate waypoint rendezvous in conjunction with multiple-threat avoidance.

All of these approaches require the numerical solution of a two-point boundary-value problem. This is quite difficult as well as computationally intensive (see [30], Chapter 7) and must be considered to be a disadvantage of such methods.

### 3.4 Collocation Methods

Collocation is a method for finding an approximate solution to integral and differential equations. Using this method, the form of the solution is specified to be some combination of a family of functions, for example a linear combination of a set of polynomials, and the solution is determined from the condition that the equations be satisfied at certain given points called the collocation knots or collocation nodes. This kind of approach, involving approximating the solution and then solving the approximate problem numerically is known as a direct approach.

In [34], Hargraves and Paris of Boeing presented a trajectory optimisation method which represented state and control variables by piecewise polynomials. The control problem was converted to a nonlinear programming problem, which was solved numerically, using a quadratic programming package. The main advantage of this method was that it made it easier to generalise the problem (from the original optimal control problem) to include such features as path constraints, discontinuous states and control inequalities. This work was applied to the design of a minimum-time climb intercept manoeuvre.

This type of approach and the relationship between the original optimal control problem and the approximating nonlinear programming problem were examined by Enright and Conway [35]. They also proposed an alternative scheme which they demonstrated on some examples involving spacecraft.

Jansch and Paus [36] extended the approach by introducing movable collocation nodes

placed optimally by the algorithm, in order to reduce their number. This was demonstrated on some aircraft trajectory generation problems. It was shown that the solution quality could be improved with little increase in computation. Examples of flight paths which were required to avoid obstacles were presented.

The "dynamic interpolating splines" described by Jackson and Crouch [12] appear to be a special case of the collocation approach. A collocation method would appear to be capable of finding an approximate solution to the problems formulated using control theoretic approaches [28, 31, 32, 33].

### **3.5 Artificial Intelligence Based Algorithms**

#### **3.5.1 Artificial Neural Nets**

Artificial neural networks (ANNs) are structures consisting of interconnected processing elements (neurons). These processing elements may be implemented in software or in hardware. The most widely used ANN is the feedforward network (FFN), which is usually trained by backpropagation. During this training, the network is presented with a set of training inputs and desired outputs. It attempts to make its outputs as close as possible (often in a least squares sense) to the desired outputs by varying the values of weights associated with each neuron. (The errors between the actual and desired outputs can be thought of as propagating back through the network to influence the values of the weights.) Once the FFN has been trained, its operation involves generating outputs in response to the inputs with which it is presented. These will usually not be elements from the training set, but hopefully, the network will have "learned desirable behaviour" so that the outputs it generates will be appropriate.

Burgin and Schnetzler [37] examined the applicability of ANNs to flight control augmentation and to a simple route planner, capable of solving a travelling-salesman-like problem.

In [38], Reibling posed the problem of generating an optimal aircraft trajectory giving minimum total threat exposure in two ways as problems in mathematical physics. The first way was to find the distribution of current flow and the second was to find the propagation of wavefronts, both through inhomogeneous media. ANN's were used to compute the solutions. In this work, the weights were determined from the problem setup and the networks were used only for their parallel architecture, and not for any learning process. In effect they implemented parallel finite element computations.

#### **3.5.2 Simulated Annealing**

Simulated annealing is a numerical technique for finding satisficing solutions to optimisation problems. The behaviour of the simulated annealing algorithm in

minimising a cost function is analogous to the slow cooling of a crystalline structure. We start with a feasible (satisficing) solution and stochastically perturb it in such a way that the new perturbed solution is also feasible. If the cost function evaluated at the new perturbed solution is smaller than at the unperturbed solution, the perturbed solution is accepted as a new successor solution and the process is repeated by perturbing the new solution. In order to allow the algorithm to avoid getting trapped in local minima, a perturbed solution which gives a larger cost may also be accepted as a successor. The probability of accepting as a successor a larger cost solution is reduced as the algorithm proceeds.

Jackson and McDowell [39] proposed a modification to the standard algorithm described above and applied it to examples which included an aircraft routing problem related to the travelling-salesman problem. The modification involved the use of a "dynamic" perturbation function, that is one which varied during the optimization process. The idea was that initially, large perturbations are required to avoid being trapped in local minima far from the global minimum, and that near the end of the process, small perturbations are more appropriate as only fine tuning of the solution is required. The demonstration of the concept on the travelling-salesman problem indicated that a lower cost solution could be obtained compared with that using the standard algorithm for the same number of iterations.

### 3.5.3 Genetic Search Algorithms

Genetic search algorithms, described by Goldberg [40] and Grefenstette [41], are another type of algorithm which is used for obtaining satisficing solutions to optimisation problems. They operate by performing a "guided random search" using ideas from evolution theory. The algorithms involve a population of solutions evolving towards the final result. The probability of a solution propagating depends on its fitness (cost). At each step of such an algorithm, a new generation of solutions is obtained by stochastically performing operations such as crossover, which combines elements of two solutions into a single solution, and mutation, which involves perturbing solutions, on a population of solutions distributed according to its fitness.

Nygard [42] applied a multiparadigm approach involving neural nets, genetic search and combinatorial optimisation methods to the solution of routing and scheduling problems. Some of this was applied to the routing and mission planning of an autonomous vehicle such as a cruise missile.

## 4. CONCLUSION

Techniques for the automated design of flight trajectories for low-flying military aircraft have been reviewed. Conclusions regarding the suitability of various optimisation

algorithms considered for the terrain avoidance/terrain following problem are given here.

Of the two methods, DP and  $A^*$ , based on combinatorial search, it appears that DP offers no advantages over the  $A^*$  algorithm. Both methods are able to find a global optimum in a discretised search space, but  $A^*$  offers a more rapidly obtained solution as well as the possibility of trading off optimality against computation speed. Both algorithms offer great flexibility in the problem setup since they are based on computing and comparing numerical values of user supplied cost functions. The main disadvantage of these methods is that, if fine discretisation is used, the search space can become very large for problems involving long flight paths. Improvements in computer technology continue to increase the range of practical application of the methods, although it remains necessary to reduce the search space. Various schemes, including those in [20], [21] and [25] have been proposed for this purpose.

Methods [31-33] based on variational calculus do not require such large computer memory resources, but require the numerical solution of a two-point boundary-value problem. This is quite difficult as well as computationally intensive and problems involving convergence of solutions have been reported. There is also less flexibility in setting up the problem than with the combinatorial based methods.

Collocation based methods [34-36] allow a more flexible problem setup than variational calculus based methods. Their reliance on numerical optimisation routines, usually quadratic programming, does not give as much flexibility as the combinatorial based methods.

The results reported to date [37, 38] using methods based on artificial neural nets to compute trajectories do not appear very encouraging.

It is difficult to evaluate the worth of simulated annealing algorithms for this application from the work [39] located. They do, however appear to be less powerful than genetic algorithms [40, 41], which seem to offer the prospect of impressive performance [42]. With both schemes, however, there is no guarantee of finding an optimal solution and the ideas behind their operation are largely empirical. The problem setup appears to be quite flexible.

Most recent trajectory generation work at ARL [22-25] has been based on the  $A^*$  algorithm which offers a simply obtained and understood result which may be refined. There is thus a significantly lower technical risk associated with this method than with the others reviewed.

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